Short Papers

Modeling the Gate I/V Characteristic of a GaAs MESFET for Volterra-Series Analysis

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Abstract —**This paper shows that the Taylor-series coefficients of a FET's gate/drain I/V characteristic, which is used to model this nonlinearity for Volterra-series analysis, can be derived from low-frequency RF measurements of harmonic output levels. The method circumvents many of the problems of using dc measurements to characterize this nonlinearity.**

I. Introduction

Volterra-series analysis [1]–[3] is an efficient and practical method for determining intermodulation levels in small-signal MESFET amplifiers. The Volterra series is particularly valuable when implemented in a general-purpose computer program [4], because it can be used to analyze very large or complex circuits, does not require that the circuit topology be simplified, and is significantly more efficient than time-domain or harmonic-balance methods.

Although much research has been directed at developing large-signal FET models for use with harmonic-balance analysis, very little work has been done to model GaAs MESFET's for Volterra-series analysis. Modeling GaAs MESFET's presents a number of subtle problems that have been noted occasionally by other authors (see, e.g., [5] and [6]). One of the most serious of these problems is the difficulty of modeling the FET's "nonlinear transconductance," or, more precisely, its nonlinear incremental gate/drain I/V characteristics. This paper describes a method for determining the parameters of this characteristic by means of simple, low-frequency RF measurements.

II. The MESFET Model

Fig. 1 shows a small-signal nonlinear model of a MESFET. It consists of a modified conventional small-signal linear model in which three elements—the gate/source capacitance C_{gs}, the drain/source resistance R_{ds}, and the drain current $i_d(v_g)$—are nonlinear. Although C_{gs} and R_{ds} are significant sources of intermodulation in small-signal FET's, in most cases the dominant nonlinearity is that of $i_d(v_g)$.

The nonlinear resistive elements are characterized by Taylor-series expansions of their I/V characteristics in the vicinity of the dc bias point. Thus, the gate/drain transfer nonlinearity $i_d(v_g)$ is

$$i_d = \left.\frac{dI_d(V_g)}{dV_g}\right|_{V_g=V_{g,0}} v_g + \frac{1}{2}\left.\frac{d^2I_d(V_g)}{dV_g^2}\right|_{V_g=V_{g,0}} v_g^2 + \frac{1}{6}\left.\frac{d^3I_d(V_g)}{dV_g^3}\right|_{V_g=V_{g,0}} v_g^3 + \cdots$$

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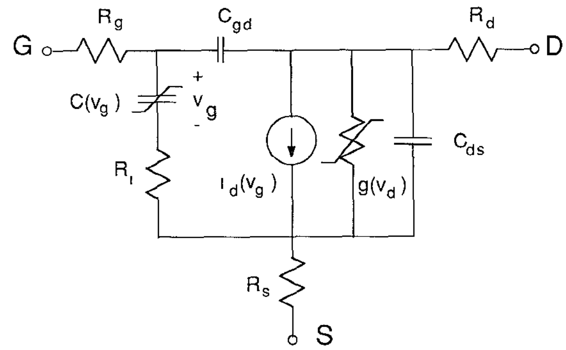

Fig. 1. Small-signal, nonlinear equivalent circuit of a GaAs MESFET

where $I_d(V_g)$ is the large-signal gate/drain characteristic and i_d and v_g are, respectively, the incremental (RF) gate and drain currents, i.e., the current and voltage deviations from the bias point $I_d(V_{g,0})$. Then i_d is expressed as

$$i_d = a_1 v_g + a_2 v_g^2 + a_3 v_g^3 + \cdots. \quad (2)$$

The traditional method of determining the series coefficients a_n is to measure the I/V characteristic at a fixed drain-bias voltage and to perform a least-squares fit of $i_d(v_g)$ to a polynomial of the desired degree. Although this process is usually adequate for determining a_1 (which is equivalent to the linear transconductance), it is often unsatisfactory for determining the higher order coefficients. Because of the ill-conditioned nature of the normal equation, the values of a_n determined in this manner are very sensitive to measurement inaccuracy, round-off errors, and the selection of data points.

Differentiating the measured data directly is often adequate for determining a_1 and sometimes a_2; however, differentiation exacerbates the effects of round-off errors and thus makes the higher derivatives unreliable.

A more fundamental problem is related to traps in the FET's channel; these introduce long time constants into $i_d(v_g)$ and cause differences between the dc and RF I/V characteristics. Trapping effects are often not evident in a_1 or a_2, but derivatives of higher order are very sensitive to them. This is particularly the case when automated equipment is used to measure $I_d(V_g)$; differences in stepping speed and dwell time at each data point affect the coefficients' values strongly.

An effective way of circumventing these problems is to derive the series coefficients from RF measurements instead of dc. The RF measurements are performed at a frequency in the VHF range, at which both trapping effects and the FET's reactive parasitics are negligible. Under these conditions the levels of the harmonic output components are functions of only the input power, the FET's source resistance, and the series coefficients. It is a simple matter to measure the harmonic levels and to derive the coefficients from them.

III. Description of the Measurements

When the RF frequency is very low, the equivalent circuit of Fig. 1 can be simplified to form the circuit shown in Fig. 2. In Fig. 2 the FET is driven by an RF source at a frequency of

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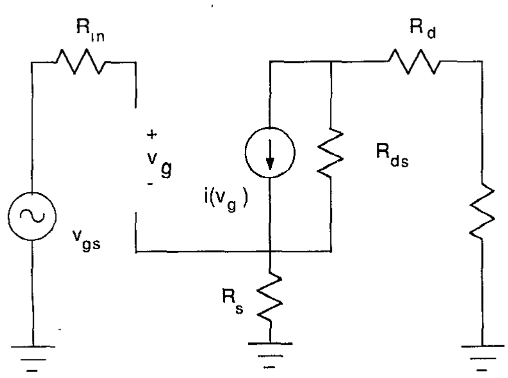

Fig. 2. Equivalent circuit of a FET valid at frequencies in the VHF range. R_L is the output-shunting resistor used to minimize harmonic generation in R_{ds}.

approximately 50 MHz, and the drain is terminated in a resistance R_L, with $R_L \ll R_{ds}$. R_L is a shunting resistor that reduces the current in R_{ds} to the point where its effects are negligible; R_L should be made as small as possible while still allowing harmonic output components to be observable on a spectrum analyzer. Because the series coefficients are derived from the relative levels of the harmonics, and not their absolute powers, the value of R_L does not affect the accuracy of the measurement.

Using the method of nonlinear currents [2], [3], one can show that the first three harmonics of the drain current, I_{1-3}, are

$$I_1 = \frac{a_1 V_{gs}}{1 + a_1 R_s} \tag{3}$$

$$I_2 = \frac{a_2(1 - a_1 R_s) V_{gs}^2}{2(1 + a_1 R_s)^2} \tag{4}$$

$$I_3 = \frac{(a_3 - 2a_2^2 R_s)(1 - a_1 R_s) V_{gs}^3}{4(1 + a_1 R_s)^3}. \tag{5}$$

V_{gs} is the magnitude of $v_{gs}(t)$; i.e.,

$$v_{gs}(t) = V_{gs}\cos(\omega t) \tag{6}$$

and the available power of the source is

$$P_a = \frac{V_{gs}^2}{8R_{in}}. \tag{7}$$

Substituting (7) into (3) and (4), dividing (4) by (3), and squaring the result gives the ratio of second-harmonic output power to fundamental output power, IMR_2:

$$\mathrm{IMR}_2 = \frac{2a_2^2 R_{in}(1 - a_1 R_s)^2 P_a}{a_1^2(1 + a_1 R_s)^2}. \tag{8}$$

A similar process gives the ratio of third-harmonic output power to fundamental output power, IMR_3:

$$\mathrm{IMR}_3 = \left[\frac{2(a_3 - 2a_2^2 R_s)(1 - a_1 R_s) R_{in}}{a_1(1 + a_1 R_s)^2}\right]^2 P_a^2. \tag{9}$$

When the FET is biased and excited by a low-frequency RF source, the values of a_n can be found by measuring the harmonic output power and solving (7) through (9). The process is as follows:

1) a_1 is determined in the conventional manner, i.e., by dc or RF measurements;
2) IMR_2 is measured by means of a spectrum analyzer, and (8) is solved to determine a_2;
3) IMR_3 is measured in similar fashion and (9) is solved to determine a_3.

Equation (8) does not indicate whether a_2 is positive or negative. However, because a MESFET's transconductance invariably rises with v_g, a_2 is invariably positive (this is generally not the case in HEMT's, but even then the regions of positive and negative a_2 are clear from the transconductance curve). Because of the squared term in brackets, two values of a_3 satisfy (9). One can determine the correct root by measuring a few values of a_2 at nearby bias points and picking the value of a_3 that most closely matches $a_3 = 0.33\, da_2/dv_g$. Usually the two values of a_3 have different signs; then, to select the correct value of a_3, one need only determine whether a_2 rises or falls with v_g.

TABLE I
TAYLOR-SERIES COEFFICIENTS
AVANTEK AT10650-5

I_d	a_1	a_2	a_3
0.019	0.041	0.0188	-0.0148
0.020	0.041	0.0171	-0.0145
0.021	0.041	0.0158	-0.0128

IV. EXPERIMENTAL RESULTS

This method was used to determine the incremental gate I/V characteristic of a packaged Avantek AT10650-5 MESFET biased at a drain voltage of 3 V and drain current of 20 mA. The FET's transconductance was measured at dc, and its small-signal equivalent circuit (including the package parasitics) was determined by adjusting its circuit element values until good agreement between calculated and measured S parameters was obtained. The FET was then installed in a low-frequency test fixture having a shunting resistance R_L of 5 Ω. The input power P_a was −11 dBm at a frequency of 50 MHz.

The a_n coefficients were determined at 19, 20, 21 mA, and the values in Table I were obtained. The coefficient a_2 was found to decrease with I_d, and the two roots of (9) had different signs; therefore the negative value of a_3 is the correct one.

The FET's gate/source capacitance was modeled as an ideal Schottky-barrier junction capacitance, and the nonlinear drain/source resistance was derived from low-frequency Y parameters measured at a variety of bias voltages. When the source and load terminations were 50 Ω, the program described in [4] predicted the FET's third-order intermodulation intercept point to be 20.8 dBm at 10 GHz. This result agrees well with the measured value of the intercept point, which was 22.4 dBm. When the output port was conjugate matched, the calculated intercept point was 24.1 dBm, compared to a measured value of 23.0 dBm. Other tests have shown equal or superior agreement. These results imply that the intermodulation levels in small-signal amplifiers can be predicted within 2 to 3 dB. We believe that the major factor that limits the accuracy is the linear part of the FET's equivalent-circuit model, especially the part that models the package. Thus, even better accuracy could be obtained with chip or MMIC circuits.

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Note on the Impedance of a Wire Grid Parallel to a Homogeneous Interface

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Abstract—We provide new numerical data for the correction factor which is used to calculate the impedance of a planar wire grid parallel to the interface between two dielectric half-spaces. Comparisons are made with earlier investigations which clarify, extend, and supersede previous computations. Here we show more clearly the significant influence of the interface on the equivalent grid impedance.

I. Introduction

A wire grid over a half-space is analogous to a transmission line with a shunt impedance. This shunt impedance consists of a logarithmic term modified by a correction factor, Δ. Depending on the value of h, the height of the grid above the interface, and d, the interwire spacing, this term may have a significant contribution to the total shunt impedance.

Previous results [1], [2] give only limited numerical data for Δ. We have found that these results have a few computational and drafting errors. We have reviewed the results in [1] and [2] and present here some graphs that illustrate important features not demonstrated in the earlier papers.

II. Formulation

With respect to a Cartesian coordinate system, the wire grid is contained in the plane $x = h$ and is parallel to a plane interface at $x = 0$. The grid is composed of an array of infinite wires parallel to the z axis and spaced a distance d between centers. The wires are taken to be of circular cross section and the diameter, $2a$, is assumed to be small compared to d. The media in both half-spaces are homogeneous and lossless with permittivity ϵ_1 for $x > 0$ and permittivity ϵ_2 for $x < 0$. The magnetic permeability is assumed to be the free-space value, μ_0, everywhere. A plane wave whose electric field is parallel to the z axis impinges on the grid with an incident angle of θ_0 as indicated in Fig. 1. Under the constraints of the thin wire approximation, the

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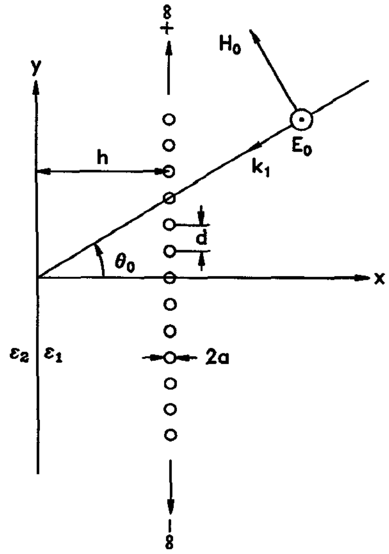

Fig. 1 The wire grid parallel to a half-space.

currents on each wire are assumed to be axially directed; from this we require that $\omega\sqrt{\mu_0\epsilon_1}\, a \ll 1$.

From previous analysis [2] it shown for this case that the equivalent shunt impedance, Z_g, is given by

$$Z_g = \frac{i\omega\mu_0 d}{2\pi}\left(\ln\frac{d}{2\pi a} + \Delta\right) + Z_w d. \tag{1}$$

Here Z_w is the axial impedance of the wire and Δ is the correction factor. The axial impedance can be expressed in terms of the modified Bessel functions as follows [3]:

$$Z_w = \frac{\eta_w}{2\pi a}\frac{I_0(\gamma_w a)}{I_1(\gamma_w a)} \tag{2}$$

where

$$\gamma_w = \sqrt{i\omega\mu_w(\sigma_w + i\omega\epsilon_w)} \tag{3}$$

and

$$\eta_w = \sqrt{\frac{i\omega\mu_w}{(\sigma_w + i\epsilon_w\omega)}}\,. \tag{4}$$

Also from [2], the correction term, Δ, is shown to be

$$\Delta = \frac{1}{2}\sum_{m=1}^{\infty}\left[\frac{1 + R_m \exp\left[-4\pi|h|d^{-1}\sqrt{(m + D_1\sin\theta_0)^2 - D_1^2}\right]}{\sqrt{(m + D_1\sin\theta_0)^2 - D_1^2}} + \frac{1 + R_{-m}\exp\left[-4\pi|h|d^{-1}\sqrt{(m - D_1\sin\theta_0)^2 - D_1^2}\right]}{\sqrt{(m - D_1\sin\theta_0)^2 - D_1^2}} - \frac{2}{m}\right] \tag{5}$$

where

$$R_m = \frac{\sqrt{(m + D_1\sin\theta_0)^2 - D_1^2} - \sqrt{(m + D_1\sin\theta_0)^2 - D_2^2}}{\sqrt{(m + D_1\sin\theta_0)^2 - D_1^2} + \sqrt{(m + D_1\sin\theta_0)^2 - D_2^2}} \tag{6}$$

and R_{-m} is obtained by replacing m with $-m$. We also have made use of the normalized dimensions D_1 and D_2, where $D_1 = d/\lambda_1$ and $D_2 = d/\lambda_2$, λ_1 and λ_2 being the wavelengths in the respective half-spaces. Here we should note that $D_1 =$